

Latitude

This article is about the geographical reference system. For other uses, see [Latitude \(disambiguation\)](#).

In geography, **latitude** (φ) is a geographic coordinate



*A graticule on the Earth as a sphere or an ellipsoid. The lines from pole to pole are lines of constant longitude, or **meridians**. The circles parallel to the equator are lines of constant latitude, or **parallels**. The graticule determines the latitude and longitude of points on the surface.*

that specifies the north-south position of a point on the Earth's surface. Latitude is an angle (defined below) which ranges from 0° at the Equator to 90° (North or South) at the poles. Lines of constant latitude, or **parallels**, run east-west as circles parallel to the equator. Latitude is used together with longitude to specify the precise location of features on the surface of the Earth. Two levels of abstraction are employed in the definition of these coordinates. In the first step the physical surface is modelled by the *geoid*, a surface which approximates the mean sea level over the oceans and its continuation under the land masses. The second step is to approximate the geoid by a mathematically simpler reference surface. The simplest choice for the reference surface is a *sphere*, but the geoid is more accurately modelled by an *ellipsoid*. The definitions of latitude and longitude on such reference surfaces are detailed in the following sections. Lines of constant latitude and longitude together constitute a *graticule* on the reference surface. The latitude of a point on the *actual* surface is that of the corresponding point on the reference surface, the correspondence being along the *normal* to the reference surface

which passes through the point on the physical surface. Latitude and longitude together with some specification of height constitute a *geographic coordinate system* as defined in the specification of the ISO 19111 standard.^[1]

Since there are many different reference ellipsoids the latitude of a feature on the surface is not unique: this is stressed in the ISO standard which states that “without the full specification of the coordinate reference system, coordinates (that is latitude and longitude) are ambiguous at best and meaningless at worst”. This is of great importance in accurate applications, such as GPS, but in common usage, where high accuracy is not required, the reference ellipsoid is not usually stated.

In English texts the latitude angle, defined below, is usually denoted by the Greek lower-case letter phi (φ or ϕ). It is measured in degrees, minutes and seconds or decimal degrees, north or south of the equator.

Measurement of latitude requires an understanding of the gravitational field of the Earth, either for setting up *theodolites* or for determination of GPS satellite orbits. The study of the *figure of the Earth* together with its gravitational field is the science of *geodesy*. These topics are not discussed in this article. (See for example the textbooks by Torge^[2] and Hofmann-Wellenhof and Moritz.)^[3]

This article relates to coordinate systems for the Earth: it may be extended to cover the Moon, planets and other celestial objects by a simple change of nomenclature.

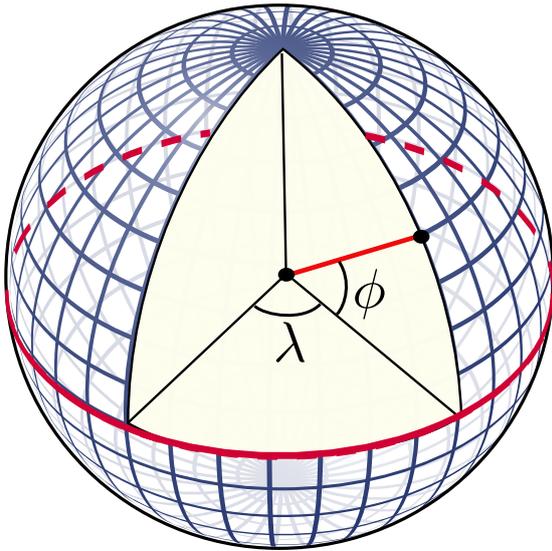
The following lists are available:

- [List of cities by latitude](#)
- [List of countries by latitude](#)

1 Latitude on the sphere

1.1 The graticule on the sphere

The graticule formed by the lines of constant latitude and constant longitude is constructed with reference to the rotation axis of the Earth. The primary reference points are the poles where the axis of rotation of the Earth intersects the reference surface. Planes which contain the rotation axis intersect the surface in the *meridians* and the angle between any one meridian plane and that through Greenwich (the *Prime Meridian*) defines the longitude: meridians are lines of constant longitude. The plane through the

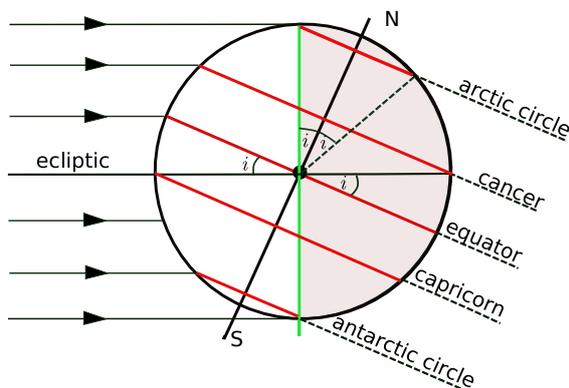


A perspective view of the Earth showing how latitude (φ) and longitude (λ) are defined on a spherical model. The graticule spacing is 10 degrees.

centre of the Earth and orthogonal to the rotation axis intersects the surface in a great circle called the equator. Planes parallel to the equatorial plane intersect the surface in circles of constant latitude; these are the parallels. The equator has a latitude of 0° , the North Pole has a latitude of 90° north (written 90° N or $+90^\circ$), and the South Pole has a latitude of 90° south (written 90° S or -90°). The latitude of an arbitrary point is the angle between the equatorial plane and the radius to that point.

The latitude that is defined in this way for the sphere is often termed the spherical latitude to avoid ambiguity with auxiliary latitudes defined in subsequent sections.

1.2 Named latitudes



The orientation of the Earth at the December solstice.

Besides the equator, four other parallels are of significance:

The plane of the Earth's orbit about the sun is called the **ecliptic**. The plane perpendicular to the rotation axis of the Earth is the equatorial plane. The angle between the ecliptic and the equatorial plane is called the inclination of the ecliptic, denoted by i in the figure. The current value of this angle is $23^\circ 26' 21''$.^[4] It is also called the **axial tilt** of the Earth since it is equal to the angle between the axis of rotation and the normal to the ecliptic.

The figure shows the geometry of a cross section of the plane normal to the ecliptic and through the centres of the Earth and the Sun at the December solstice when the sun is overhead at some point of the **Tropic of Capricorn**. The south polar latitudes below the **Antarctic Circle** are in daylight whilst the north polar latitudes above the **Arctic Circle** are in night. The situation is reversed at the June solstice when the sun is overhead at the **Tropic of Cancer**. The latitudes of the tropics are equal to the inclination of the ecliptic and the polar circles are at latitudes equal to its complement. Only at latitudes in between the two tropics is it possible for the sun to be directly overhead (at the **zenith**).

The named parallels are clearly indicated on the Mercator projections shown below.

1.3 Map projections from the sphere

On **map projections** there is no simple rule as to how meridians and parallels should appear. For example, on the spherical **Mercator projection** the parallels are horizontal and the meridians are vertical whereas on the **Transverse Mercator projection** there is no correlation of parallels and meridians with horizontal and vertical; both are complicated curves. The red lines are the named latitudes of the previous section.

For map projections of large regions, or the whole world, a spherical Earth model is completely satisfactory since the variations attributable to ellipticity are negligible on the final printed maps.

1.4 Meridian distance on the sphere

On the sphere the normal passes through the centre and the latitude (φ) is therefore equal to the angle subtended at the centre by the meridian arc from the equator to the point concerned. If the meridian distance is denoted by $m(\varphi)$ then

$$m(\varphi) = \frac{\pi}{180} R \varphi_{\text{degrees}} = R \varphi_{\text{radians}}.$$

where R denotes the **mean radius** of the Earth. R is equal to 6,371 km or 3,959 miles. No higher accuracy is appropriate for R since higher precision results necessitate an ellipsoid model. With this value for R the meridian

length of 1 degree of latitude on the sphere is 111.2 km or 69 miles. The length of 1 minute of latitude is 1.853 km, or 1.15 miles. (See [nautical mile](#)).

2 Latitude on the ellipsoid

2.1 Ellipsoids

In 1687 Isaac Newton published the *Philosophiæ Naturalis Principia Mathematica* in which he proved that a rotating self-gravitating fluid body in equilibrium takes the form of an oblate ellipsoid.^[5] (This article uses the term ellipsoid in preference to the older term *spheroid*.) Newton's result was confirmed by geodetic measurements in the eighteenth century. (See [Meridian arc](#).) An oblate ellipsoid is the three-dimensional surface generated by the rotation of an ellipse about its shorter axis (minor axis). "Oblate ellipsoid of revolution" is abbreviated to **ellipsoid** in the remainder of this article. (Ellipsoids which do not have an axis of symmetry are termed tri-axial.)

Many different [reference ellipsoids](#) have been used in the history of [geodesy](#). In pre-satellite days they were devised to give a good fit to the [geoid](#) over the limited area of a survey but, with the advent of [GPS](#), it has become natural to use reference ellipsoids (such as [WGS84](#)) with centres at the centre of mass of the Earth and minor axis aligned to the rotation axis of the Earth. These geocentric ellipsoids are usually within 100m of the geoid. Since latitude is defined with respect to an ellipsoid, the position of a given point is different on each ellipsoid: one can't exactly specify the latitude and longitude of a geographical feature without specifying the ellipsoid used. Many maps maintained by national agencies are based on older ellipsoids so it is necessary to know how the latitude and longitude values are transformed from one ellipsoid to another. GPS handsets include software to carry out datum transformations which link WGS84 to the local reference ellipsoid with its associated grid.

2.2 The geometry of the ellipsoid

The shape of an ellipsoid of revolution is determined by the shape of the ellipse which is rotated about its minor (shorter) axis. Two parameters are required. One is invariably the equatorial radius, which is the semi-major axis, a . The other parameter is usually (1) the polar radius or semi-minor axis, b ; or (2) the (first) flattening, f ; or (3) the eccentricity, e . These parameters are not independent: they are related by

$$f = \frac{a - b}{a}, \quad e^2 = 2f - f^2, \quad b = a(1 - f) = a\sqrt{1 - e^2}$$

Many other parameters (see [ellipse](#), [ellipsoid](#)) appear in the study of geodesy, geophysics and map projections but

they can all be expressed in terms of one or two members of the set a, b, f and e . Both f and e are small and often appear in series expansions in calculations; they are of the order $1/300$ and 0.08 , respectively. Values for a number of ellipsoids are given in [Figure of the Earth](#). Reference ellipsoids are usually defined by the semi-major axis and the *inverse* flattening, $1/f$. For example, the defining values for the [WGS84](#) ellipsoid, used by all [GPS](#) devices, are^[6]

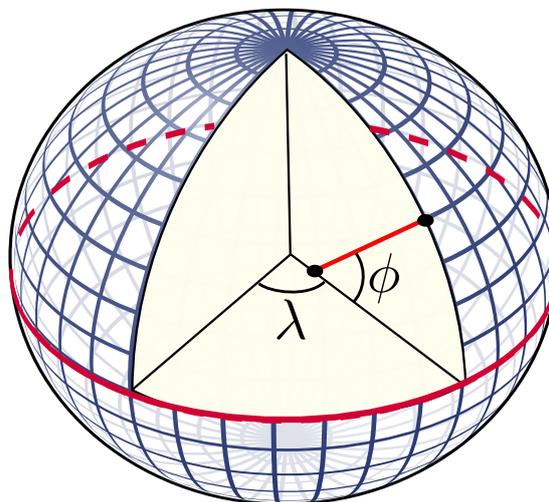
- a (equatorial radius): 6,378,137.0 m exactly
- $1/f$ (inverse flattening): 298.257,223,563 exactly

from which are derived

- b (polar radius): 6,356,752.3142 m
- e^2 (eccentricity squared): 0.006,694,379,990,14

The difference of the major and minor semi-axes is about 21 km and as fraction of the semi-major axis it equals the flattening; on a computer the ellipsoid could be sized as 300px by 299px. This would be indistinguishable from a sphere shown as 300px by 300px, so illustrations always exaggerate the flattening.

2.3 Geodetic and geocentric latitudes



The definition of geodetic latitude (ϕ) and longitude (λ) on an ellipsoid. The normal to the surface does not pass through the centre, except at the equator and at the poles.

The graticule on the ellipsoid is constructed in exactly the same way as on the sphere. The normal at a point on the surface of an ellipsoid does not pass through the centre, except for points on the equator or at the poles, but the definition of latitude remains unchanged as the

angle between the normal and the equatorial plane. The terminology for latitude must be made more precise by distinguishing

Geodetic latitude: the angle between the normal and the equatorial plane. The standard notation in English publications is φ . This is the definition assumed when the word latitude is used without qualification. The definition must be accompanied with a specification of the ellipsoid.

Geocentric latitude: the angle between the radius (from centre to the point on the surface) and the equatorial plane. (Figure below). There is no standard notation: examples from various texts include ψ , q , φ' , φ_c , φ_g . This article uses ψ .

Spherical latitude: the angle between the normal to a spherical reference surface and the equatorial plane.

Geographic latitude must be used with care. Some authors use it as a synonym for geodetic latitude whilst others use it as an alternative to the astronomical latitude.

Latitude (unqualified) should normally refer to the geodetic latitude.

The importance of specifying the reference datum may be illustrated by a simple example. On the reference ellipsoid for WGS84, the centre of the Eiffel Tower has a geodetic latitude of $48^\circ 51' 29''$ N, or 48.8583° N and longitude of $2^\circ 17' 40''$ E or 2.2944° E. The same coordinates on the datum ED50 define a point on the ground which is 140 m distant from Tower. A web search may produce several different values for the latitude of the Tower; the reference ellipsoid is rarely specified.

2.4 Length of a degree of latitude

Main article: [Meridian arc](#)

In [Meridian arc](#) and standard texts^{[2][7][8]} it is shown that the distance along a meridian from latitude φ to the equator is given by (φ in radians)

$$m(\phi) = \int_0^\phi M(\phi) d\phi = a(1-e^2) \int_0^\phi (1 - e^2 \sin^2 \phi)^{-3/2} d\phi$$

The function $M(\phi)$ in the first integral is the meridional radius of curvature.

The distance from the equator to the pole is

$$m_p = m(\pi/2)$$

For [WGS84](#) this distance is 10,001.965729 km.

The evaluation of the meridian distance integral is central to many studies in geodesy and map projection. It can be evaluated by expanding the integral by the binomial series and integrating term by term: see [Meridian arc](#) for details. The length of the meridian arc between two given latitudes is given by replacing the limits of the integral by the latitudes concerned. The length of a *small* meridian arc is given by^{[7][8]}

$$\delta m(\phi) = M(\phi) \delta\phi = a(1 - e^2) (1 - e^2 \sin^2 \phi)^{-3/2} \delta\phi$$

When the latitude difference is 1 degree, corresponding to $\pi/180$ radians, the arc distance is about

$$\Delta_{\text{LAT}}^1 = \frac{\pi a(1 - e^2)}{180(1 - e^2 \sin^2 \phi)^{3/2}}$$

The distance in metres (correct to 0.01 metre) between latitudes ($\phi - 0.5$ deg) and ($\phi + 0.5$ deg) on the WGS84 spheroid is

$$\Delta_{\text{LAT}}^1 = 111132.954 - 559.822 \cos 2\phi + 1.175 \cos 4\phi$$

The variation of this distance with latitude (on WGS84) is shown in the table along with the [length of a degree of longitude](#):

$$\Delta_{\text{LONG}}^1 = \frac{\pi a \cos \phi}{180(1 - e^2 \sin^2 \phi)^{1/2}}$$

A calculator for any latitude is provided by the U.S. government's [National Geospatial-Intelligence Agency](#) (NGA).^[9]

3 Auxiliary latitudes

There are six **auxiliary latitudes** that have applications to special problems in geodesy, geophysics and the theory of map projections:

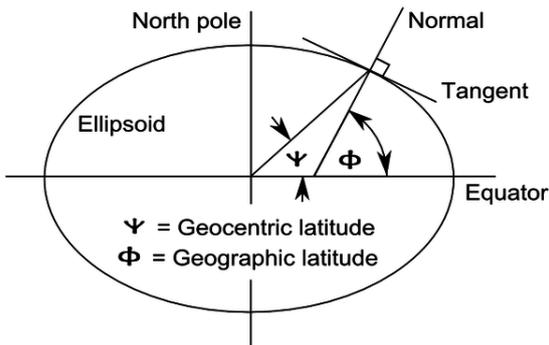
- Geocentric latitude
- Reduced (or parametric) latitude
- Rectifying latitude
- Authalic latitude
- Conformal latitude

- Isometric latitude

The definitions given in this section all relate to locations on the reference ellipsoid but the first two auxiliary latitudes, like the geodetic latitude, can be extended to define a three-dimensional geographic coordinate system as discussed below. The remaining latitudes are not used in this way; they are used *only* as intermediate constructs in map projections of the reference ellipsoid to the plane or in calculations of geodesics on the ellipsoid. Their numerical values are not of interest. For example no one would need to calculate the authalic latitude of the Eiffel Tower.

The expressions below give the auxiliary latitudes in terms of the geodetic latitude, the semi-major axis, a , and the eccentricity, e . (For inverses see below.) The forms given are, apart from notational variants, those in the standard reference for map projections, namely “Map projections: a working manual” by J. P. Snyder.^[10] Derivations of these expressions may be found in Adams^[11] and on-line publications by Osborne^[7] and Rapp.^[8]

3.1 Geocentric latitude



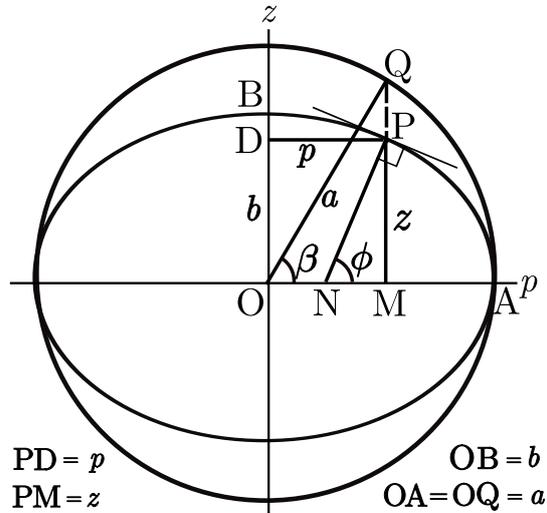
The definition of geodetic (or geographic) and geocentric latitudes.

The **geocentric latitude** is the angle between the equatorial plane and the radius from the centre to a point on the surface. The relation between the geocentric latitude (ψ) and the geodetic latitude (ϕ) is derived in the above references as

$$\psi(\phi) = \tan^{-1} [(1 - e^2) \tan \phi] .$$

The geodetic and geocentric latitudes are equal at the equator and poles. The value of the squared eccentricity is approximately 0.0067 (depending on the choice of ellipsoid) and the maximum difference of $(\phi - \psi)$ is approximately 11.5 minutes of arc at a geodetic latitude of $45^\circ 5'$.

3.2 Reduced (or parametric) latitude



Definition of the reduced latitude (β) on the ellipsoid.

The **reduced or parametric latitude**, β , is defined by the radius drawn from the centre of the ellipsoid to that point Q on the surrounding sphere (of radius a) which is the projection parallel to the Earth's axis of a point P on the ellipsoid at latitude ϕ . It was introduced by Legendre^[12] and Bessel^[13] who solved problems for geodesics on the ellipsoid by transforming them to an equivalent problem for spherical geodesics by using this smaller latitude. Bessel's notation, $u(\phi)$, is also used in the current literature. The reduced latitude is related to the geodetic latitude by:^{[7][8]}

$$\beta(\phi) = \tan^{-1} \left[\sqrt{1 - e^2} \tan \phi \right]$$

The alternative name arises from the parameterization of the equation of the ellipse describing a meridian section. In terms of Cartesian coordinates p , the distance from the minor axis, and z , the distance above the equatorial plane, the equation of the ellipse is:

$$\frac{p^2}{a^2} + \frac{z^2}{b^2} = 1.$$

The Cartesian coordinates of the point are parameterized by

$$p = a \cos \beta, \quad z = b \sin \beta;$$

Cayley suggested the term *parametric latitude* because of the form of these equations.^[14]

The reduced latitude is not used in the theory of map projections. Its most important application is in the theory of ellipsoid geodesics. (Vincenty, Karney).^[15]

3.3 Rectifying latitude

See also: Rectifying radius

The **rectifying latitude**, μ , is the meridian distance scaled so that its value at the poles is equal to 90 degrees or $\pi/2$ radians:

$$\mu(\phi) = \frac{\pi}{2} \frac{m(\phi)}{m_p}$$

where the meridian distance from the equator to a latitude ϕ is (see Meridian arc)

$$m(\phi) = a(1 - e^2) \int_0^\phi (1 - e^2 \sin^2 \phi)^{-3/2} d\phi,$$

and the length of the meridian quadrant from the equator to the pole (the polar distance) is

$$m_p = m(\pi/2).$$

Using the rectifying latitude to define a latitude on a sphere of radius

$$R = \frac{2m_p}{\pi}$$

defines a projection from the ellipsoid to the sphere such that all meridians have true length and uniform scale. The sphere may then be projected to the plane with an **equiarectangular projection** to give a double projection from the ellipsoid to the plane such that all meridians have true length and uniform meridian scale. An example of the use of the rectifying latitude is the **Equidistant conic projection**. (Snyder, Section 16).^[10] The rectifying latitude is also of great importance in the construction of the **Transverse Mercator projection**.

3.4 Authalic latitude

See also: Authalic radius

The **authalic** (Greek for same area) latitude, ξ , gives an area-preserving transformation to a sphere.

$$\xi(\phi) = \sin^{-1} \left(\frac{q(\phi)}{q_p} \right)$$

where

$$\begin{aligned} q(\phi) &= \frac{(1 - e^2) \sin \phi}{1 - e^2 \sin^2 \phi} - \frac{1 - e^2}{2e} \ln \left(\frac{1 - e \sin \phi}{1 + e \sin \phi} \right), \\ &= \frac{(1 - e^2) \sin \phi}{1 - e^2 \sin^2 \phi} + \frac{1 - e^2}{e} \tanh^{-1}(e \sin \phi), \end{aligned}$$

and

$$q_p = q(\pi/2) = 1 - \frac{1 - e^2}{2e} \ln \left(\frac{1 - e}{1 + e} \right) = 1 + \frac{1 - e^2}{e} \tanh^{-1} e,$$

and the radius of the sphere is taken as

$$R_q = a\sqrt{q_p/2}.$$

An example of the use of the authalic latitude is the **Albers equal-area conic projection**. (Snyder,^[10] Section 14).

3.5 Conformal latitude

The **conformal latitude**, χ , gives an angle-preserving (**conformal**) transformation to the sphere.

$$\begin{aligned} \chi(\phi) &= 2 \tan^{-1} \left[\left(\frac{1 + \sin \phi}{1 - \sin \phi} \right) \left(\frac{1 - e \sin \phi}{1 + e \sin \phi} \right)^e \right]^{1/2} - \frac{\pi}{2} \\ &= 2 \tan^{-1} \left[\tan \left(\frac{\phi}{2} + \frac{\pi}{4} \right) \left(\frac{1 - e \sin \phi}{1 + e \sin \phi} \right)^{e/2} \right] - \frac{\pi}{2} \\ &= \sin^{-1} \left[\tanh \left(\tanh^{-1}(\sin \phi) - e \tanh^{-1}(e \sin \phi) \right) \right] \\ &= \text{gd} \left[\text{gd}^{-1}(\phi) - e \tanh^{-1}(e \sin \phi) \right]. \end{aligned}$$

where $\text{gd}(x)$ is the **Gudermannian function**. (See also **Mercator projection**.) The conformal latitude defines a transformation from the ellipsoid to a sphere of *arbitrary* radius such that the angle of intersection between any two lines on the ellipsoid is the same as the corresponding angle on the sphere (so that the shape of *small* elements is well preserved). A further conformal transformation from the sphere to the plane gives a conformal double projection from the ellipsoid to the plane. This is not the only way of generating such a conformal projection. For example, the 'exact' version of the **Transverse Mercator projection** on the ellipsoid is not a double projection. (It does, however, involve a generalisation of the conformal latitude to the complex plane).

3.6 Isometric latitude

The **isometric latitude** is conventionally denoted by ψ (not to be confused with the geocentric latitude): it is used in the development of the ellipsoidal versions of the normal **Mercator projection** and the **Transverse Mercator projection**. The name “isometric” arises from the fact that at any point on the ellipsoid equal increments of ψ and longitude λ give rise to equal distance displacements along the meridians and parallels respectively. The **graticule** defined by the lines of constant ψ and constant λ , divides the surface of the ellipsoid into a mesh of squares (of varying size). The isometric latitude is zero at the equator but rapidly diverges from the geodetic latitude, tending to infinity at the poles. The conventional notation is given in Snyder (page 15):^[10]

$$\begin{aligned}\psi(\phi) &= \ln \left[\tan \left(\frac{\pi}{4} + \frac{\phi}{2} \right) \right] + \frac{e}{2} \ln \left[\frac{1 - e \sin \phi}{1 + e \sin \phi} \right] \\ &= \tanh^{-1}(\sin \phi) - e \tanh^{-1}(e \sin \phi) \\ &= \text{gd}^{-1}(\phi) - e \tanh^{-1}(e \sin \phi).\end{aligned}$$

For the *normal* Mercator projection (on the ellipsoid) this function defines the spacing of the parallels: if the length of the equator on the projection is E (units of length or pixels) then the distance, y , of a parallel of latitude ϕ from the equator is

$$y(\phi) = \frac{E}{2\pi} \psi(\phi).$$

The isometric latitude is closely related to the conformal latitude:

$$\psi(\phi) = \text{gd}^{-1} \chi(\phi).$$

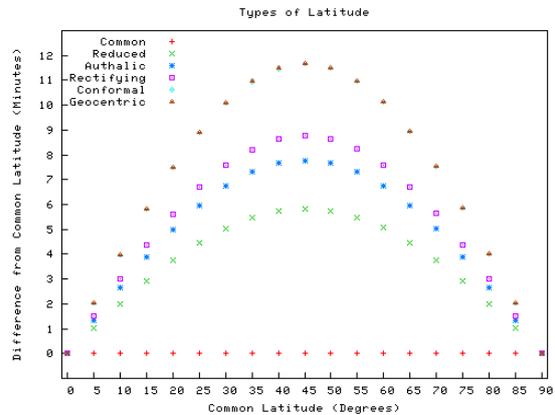
3.7 Inverse formulae and series

The formulae in the previous sections give the auxiliary latitude in terms of the geodetic latitude. The expressions for the geocentric and reduced latitudes may be inverted directly but this is impossible in the four remaining cases: the rectifying, authalic, conformal, and isometric latitudes. There are two methods of proceeding. The first is a numerical inversion of the defining equation for each and every particular value of the auxiliary latitude. The methods available are **fixed-point iteration** and **Newton-Raphson** root finding. The other, more useful, approach is to express the auxiliary latitude as a series in terms of the geodetic latitude and then invert the series by the method of **Lagrange reversion**. Such series are presented

by Adams who uses Taylor series expansions and gives coefficients in terms of the eccentricity.^[11] Osborne^[7] derives series to arbitrary order by using the computer algebra package **Maxima**^[16] and expresses the coefficients in terms of both eccentricity and flattening. The series method is not applicable to the isometric latitude and one must use the conformal latitude in an intermediate step.

3.8 Numerical comparison of auxiliary latitudes

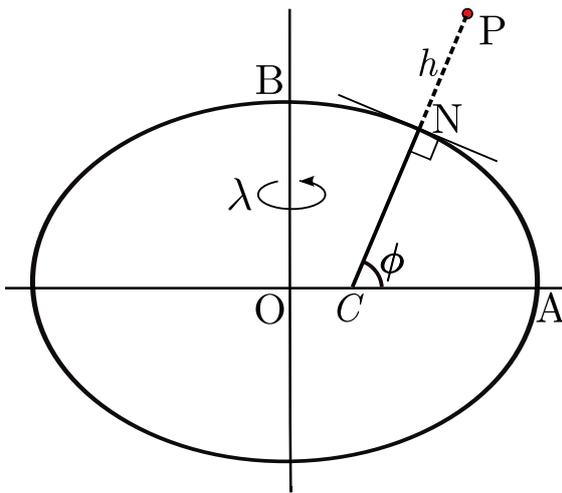
The following plot shows the magnitude of the difference between the geodetic latitude, (denoted as the “common” latitude on the plot), and the auxiliary latitudes other than the isometric latitude (which diverges to infinity at the poles). In every case the geodetic latitude is the greater. The differences shown on the plot are in arc minutes. The horizontal resolution of the plot fails to make clear that the maxima of the curves are not at 45° but calculation shows that they are within a few arc minutes of 45° . Some representative data points are given in the table following the plot. Note the closeness of the conformal and geocentric latitudes. This was exploited in the days of hand calculators to expedite the construction of map projections. (Snyder,^[10] page 108).



4 Latitude and coordinate systems

The geodetic latitude, or any of the auxiliary latitudes defined on the reference ellipsoid, constitutes with longitude a two-dimensional coordinate system on that ellipsoid. To define the position of an arbitrary point it is necessary to extend such a coordinate system into three dimensions. Three latitudes are used in this way: the geodetic, geocentric and reduced latitudes are used in geodetic coordinates, spherical polar coordinates and ellipsoidal coordinates respectively.

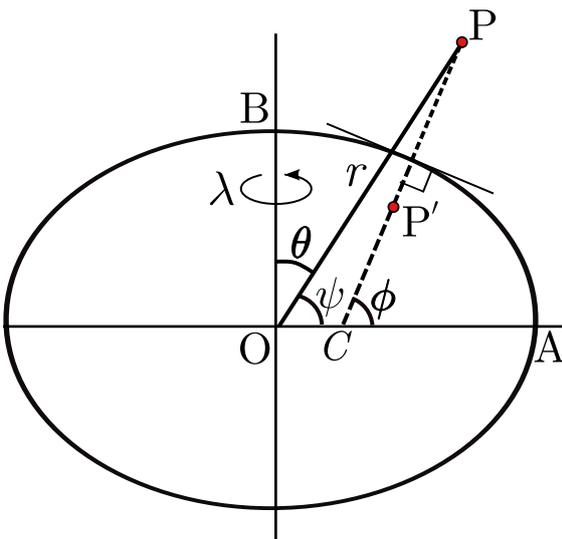
4.1 Geodetic coordinates



Geodetic coordinates $P(\phi, \lambda, h)$

At an arbitrary point P consider the line PN which is normal to the reference ellipsoid. The geodetic coordinates $P(\phi, \lambda, h)$ are the latitude and longitude of the point N on the ellipsoid and the distance PN . This height differs from the height above the geoid or a reference height such as that above mean sea level at a specified location. The direction of PN will also differ from the direction of a vertical plumb line. The relation of these different heights requires knowledge of the shape of the geoid and also the gravity field of the Earth.

4.2 Spherical polar coordinates

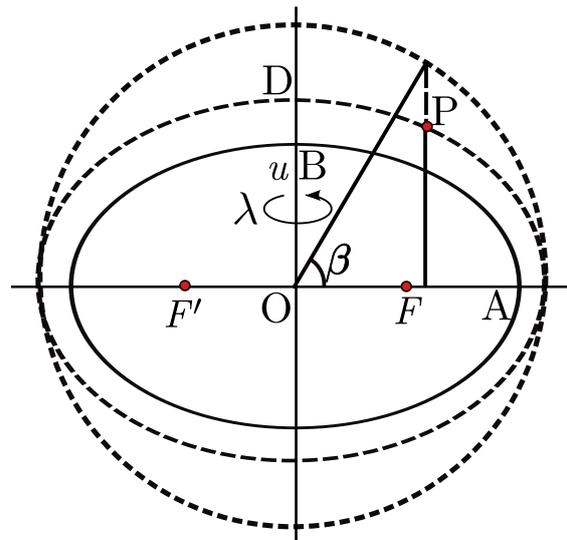


Geocentric coordinate related to spherical polar coordinates $P(r, \theta, \lambda)$

The geocentric latitude ψ is the complement of the polar angle θ in conventional spherical polar coordinates in

which the coordinates of a point are $P(r, \theta, \lambda)$ where r is the distance of P from the centre O , θ is the angle between the radius vector and the polar axis and λ is longitude. Since the normal at a general point on the ellipsoid does not pass through the centre it is clear that points on the normal, which all have the same geodetic latitude, will have differing geocentric latitudes. Spherical polar coordinate systems are used in the analysis of the gravity field.

4.3 Ellipsoidal coordinates



Ellipsoidal coordinates $P(u, \beta, \lambda)$

The reduced latitude can also be extended to a three-dimensional coordinate system. For a point P not on the reference ellipsoid (semi-axes OA and OB) construct an auxiliary ellipsoid which is confocal (same foci F, F') with the reference ellipsoid: the necessary condition is that the product ae of semi-major axis and eccentricity is the same for both ellipsoids. Let u be the semi-minor axis (OD) of the auxiliary ellipsoid. Further let β be the reduced latitude of P on the auxiliary ellipsoid. The set (u, β, λ) define the ellipsoid coordinates. (Torge^[2] Section 4.2.2). These coordinates are the natural choice in models of the gravity field for a uniform distribution of mass bounded by the reference ellipsoid.

4.4 Coordinate conversions

The relations between the above coordinate systems, and also Cartesian coordinates are not presented here. The transformation between geodetic and Cartesian coordinates may be found in Geodetic system. The relation of Cartesian and spherical polars is given in Spherical coordinate system. The relation of Cartesian and ellipsoidal coordinates is discussed in Torge.^[2]

5 Astronomical latitude

Astronomical latitude (Φ) is the angle between the equatorial plane and the true vertical at a point on the surface. The true vertical, the direction of a plumb line, is also the direction of the **gravity acceleration**, the resultant of the gravitational acceleration (mass-based) and the centrifugal acceleration at that latitude (see Torge.)^[2] Astronomic latitude is calculated from angles measured between the **zenith** and stars whose **declination** is accurately known.

In general the true vertical at a point on the surface does not exactly coincide with either the normal to the reference ellipsoid or the normal to the geoid. The angle between the astronomic and geodetic normals is usually a few seconds of arc but it is important in geodesy.^{[2][3]} The reason why it differs from the normal to the geoid is, because the geoid is an idealized, theoretical shape “at mean sea level”. Points on the real surface of the earth are usually above or below this idealized geoid surface and here the true vertical can vary slightly. Also, the true vertical at a point at a specific time is influenced by tidal forces, which the theoretical geoid averages out.

Astronomical latitude is not to be confused with **declination**, the coordinate astronomers used in a similar way to describe the locations of stars north/south of the celestial equator (see **equatorial coordinates**), nor with **ecliptic latitude**, the coordinate that astronomers use to describe the locations of stars north/south of the ecliptic (see **ecliptic coordinates**).

6 See also

- Altitude (mean sea level)
- American Practical Navigator
- Cardinal direction
- Degree Confluence Project
- Geodesy
- Geodetic system
- Geographic coordinate system
- Geographical distance
- Geotagging
- Great-circle distance
- Horse latitudes
- List of cities by latitude
- List of countries by latitude
- Longitude

- Natural Area Code
- Navigation
- Orders of magnitude (length)
- World Geodetic System

7 Footnotes

- [1] The current full documentation of ISO 19111 may be purchased from <http://www.iso.org> but drafts of the final standard are freely available at many web sites, one such is available at the following CSIRO
- [2] Torge, W (2001) *Geodesy* (3rd edition), published by De Gruyter, isbn=3-11-017072-8
- [3] Hofmann-Wellenhof, B and Moritz, H (2006). 'Physical Geodesy (second edition)' ISBN3211-33544-7.
- [4] The Astronomical Almanac published annually by the National Almanac Office in the United States (<http://asa.usno.navy.mil/>) and the United Kingdom (<http://astro.ukho.gov.uk/nao/publicat/asa.html>).
- [5] Isaac Newton:*Principia* Book III Proposition XIX Problem III, p. 407 in Andrew Motte translation, available on line at
- [6] The WGS84 parameters are listed in the National Geospatial-Intelligence Agency publication TR8350.2 page 3-1.
- [7] Osborne, P (2013)*The Mercator Projections* (Chapters 5,6)
- [8] Rapp, Richard H. (1991). *Geometric Geodesy, Part I*, Dept. of Geodetic Science and Surveying, Ohio State Univ., Columbus, Ohio.(Chapter 3)
- [9] Length of degree calculator - National Geospatial-Intelligence Agency
- [10] Snyder, John P. (1987). *Map Projections: A Working Manual*. U.S. Geological Survey Professional Paper 1395. Washington, D.C.: United States Government Printing Office. This paper can be downloaded from USGS pages.
- [11] Adams, Oscar S (1921). *Latitude Developments Connected With Geodesy and Cartography, (with tables, including a table for Lambert equal area meridional projection)*. Special Publication No. 67 of the US Coast and Geodetic Survey. A facsimile of this publication is available from the US National Oceanic and Atmospheric Administration (NOAA) at http://docs.lib.noaa.gov/rescue/cgs_specpubs/QB275U35no671921.pdf **Warning:** Adams uses the nomenclature isometric latitude for the conformal latitude of this article.
- [12] A. M. Legendre, 1806, Analyse des triangles tracés sur la surface d'un sphéroïde, Mém. de l'Inst. Nat. de France, 130—161 (1st semester).

- [13] F. W. Bessel, 1825, *Über die Berechnung der geographischen Längen und Breiten aus geodatischen Vermessungen*, *Astron.Nachr.*, 4(86), 241-254, doi:10.1002/asna.201011352, translated into English by C. F. F. Karney and R. E. Deakin as *The calculation of longitude and latitude from geodesic measurements*, *Astron. Nachr.* 331(8), 852-861 (2010), E-print arXiv:0908.1824, <http://adsabs.harvard.edu/abs/1825AN.....4..241B>.
- [14] A. Cayley, 1870, *On the geodesic lines on an oblate spheroid*, *Phil. Mag.* 40 (4th ser.), 329-340.
- [15] C. F. F. Karney (2013), *Algorithms for geodesics*, *J. Geodesy* 87(1), 43–55, DOI: 10.1007/s00190-012-0578-z.
- [16] Maxima computer algebra system

8 External links

- [GEONets Names Server](#), access to the National Geospatial-Intelligence Agency's (NGA) database of foreign geographic feature names.
- [Resources for determining your latitude and longitude](#)
- [Convert decimal degrees into degrees, minutes, seconds - Info about decimal to sexagesimal conversion](#)
- [Convert decimal degrees into degrees, minutes, seconds](#)
- [Distance calculation based on latitude and longitude - JavaScript version](#)
- [Determination of Latitude by Francis Drake on the Coast of California in 1579](#)
- [Longitude and Latitude of Points of Interest](#)
- [Online computation of all relevant quantities referring to an ellipsoidal latitude on a chosen reference ellipsoid](#)

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